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Three-body decay of nuclear resonances

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CONTENT

- 1. Theoretical formulation
- 2. Short-range interaction
- 3. Two large scattering lengths
- 4. Short-range plus Coulomb interaction
- 5. Isospin mixing
- 6. Summary and conclusions

Theoretical formulation:

Coordinates: $(\boldsymbol{x},\boldsymbol{y})$ are mass scaled Jacobi coordinates

 $\boldsymbol{r_{ij}}$ is the vector connecting particle i and j

$$x^2 = r_{ik}^2 \frac{m_i m_k}{m(m_i + m_k)}$$

$$y^2 = r_{j,ik}^2 \frac{m_j(m_i + m_k)}{m(m_i + m_k + m_j)}$$

$$ho^2 = x^2 + y^2 = \frac{1}{m(m_i + m_k + m_j)} \sum_{i < j} m_i m_j r_{ij}^2$$

$$\Omega = {\Omega_x, \Omega_y, \alpha}$$
, i.e. directions of $(\boldsymbol{x}, \boldsymbol{y})$ and $\tan \alpha = \frac{x}{y}$

Complex scaling: $\rho \to \rho \exp(i\theta)$

Adiabatic hyperspherical expansion:

Choose interactions and solve Faddeev equations for each ρ

Compute angular eigenvalues λ_n and eigenfunctions $\{\Phi_n(\rho,\Omega)\}$

The three-body bound state or resonance wave function Ψ is:

$$\Psi(\boldsymbol{x}, \boldsymbol{y}) = \Sigma_n f_n(\rho) \Phi_n(\rho, \Omega)$$

= $\Sigma_n f_n(\rho) \left(\phi_1^{(n)}(\rho, \Omega) + \phi_2^{(n)}(\rho, \Omega) + \phi_3^{(n)}(\rho, \Omega) \right)$

Solve radial equations: $f_n(\rho)$ and complex energy eigenvalues

Table 1: Components included in the three-body calculations have $K_{max} = 20$ except those specified here. The left part refers to the components in the first Jacobi set (\boldsymbol{x} connecting the two nucleons), and the right part to the ones in the second and third Jacobi sets (\boldsymbol{x} connecting the alpha-particle and one of the nucleons).

1 st Jacobi set						2^{nd} and 3^{rd} Jacobi sets						
$N \overrightarrow{\overrightarrow{x}} N$ $\overrightarrow{y} \alpha$							$\alpha \underbrace{\vec{x}}_{N} N$					
ℓ_x	ℓ_y	L	s_x	s_y	K_{max}	ℓ_x	ℓ_y	L	s_x	s_y	K_{max}	
0	2	2	0	0	180	0	2	2	1/2	0	44	
2	0	2	0	0	180	0	2	2	1/2	1	44	
1	1	1	1	1	180	2	0	2	1/2	0	70	
1	1	2	1	1	64	2	0	2	1/2	1	44	
2	2	2	0	0	90	1	1	1	1/2	1	240	
1	3	2	1	1	42	1	1	2	1/2	0	240	
3	1	2	1	1	42	1	1	2	1/2	1	44	
2	4	2	0	0	54	2	2	1	1/2	1	32	
4	2	2	0	0	54	2	2	2	1/2	0	50	
4	4	2	0	0	68	2	2	2	1/2	1	42	
						1	3	2	1/2	0	42	
						1	3	2	1/2	1	42	

Notice Jacobi coordinates

Each of the Faddeev components are partial wave expanded Rather large K_{max} in each of these many partial waves

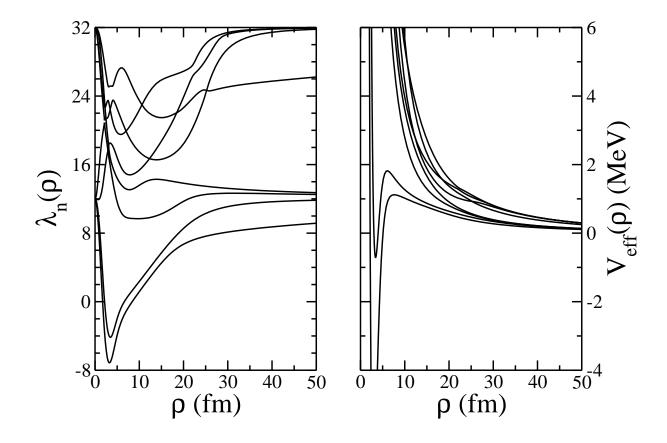


Figure 1: The real parts of the lowest 8 angular eigenvalues (left) and corresponding adiabatic potentials (right) as functions of ρ for the 2⁺ states in ⁶He (⁴He + n + n). The scaling angle is $\theta = 0.10$.

Effective hyperradial potentials Attractive region and a barrier

Resonance properties:

Energy determined by attractive pocket

Width determined by the barrier

Large-distance behavior determines final state energy distribution

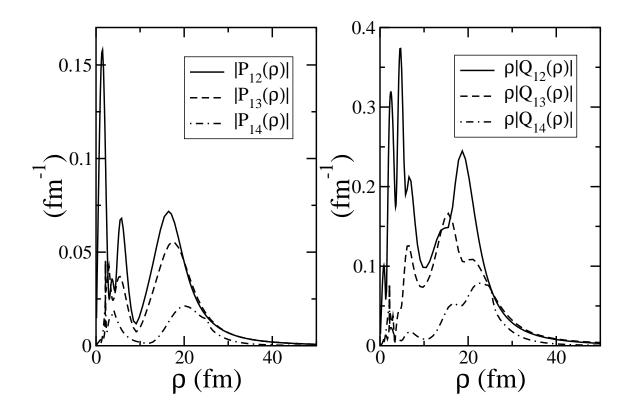


Figure 2: The coupling potentials between the four lowest adiabatic levels for $\theta = 0.10$ shown as functions of ρ for $^6\mathrm{He}(2^+)$. The first and the fourth levels have similar quantum numbers but approach the K=2 and 4 levels, respectively. To show the first (P) and second (Q) order coupling potentials in the same units (fm^{-1}) we multiply Q by ρ . (The energy unit is restored in the coupling potentials by including the omitted factor, i.e. $\hbar^2 Q/(2m)$, $\hbar^2 P/(2m)\partial/\partial\rho$).

Couplings determine relative size of radial wavefunctions Fall off at intermediate distance Numerical stability at large distance Compromise between: lowest (adiabatic) state and maintaining the structure

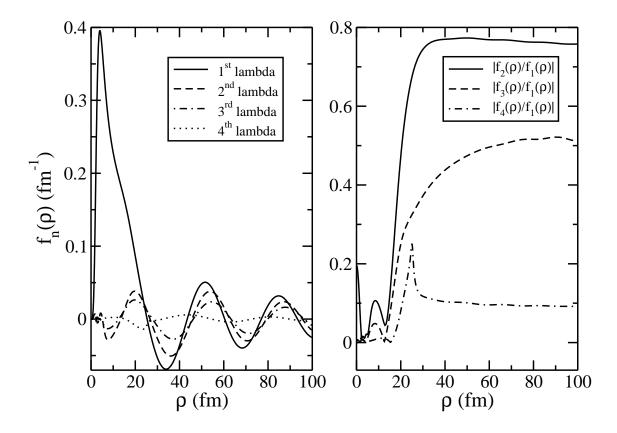
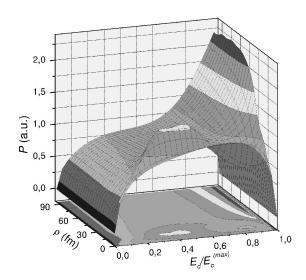


Figure 3: The lowest four radial wavefunctions (left) and their relative sizes (right) for $\theta = 0.10$ as functions ρ for the $^6{\rm He}(2^+)$ resonance.

$$\Psi(\boldsymbol{x},\boldsymbol{y}) = \Sigma_n f_n(\rho) \Phi_n(\rho,\Omega)$$

Each fall off exponentially while oscillating around zero Relative size at large distance is stable Determine energy distribution



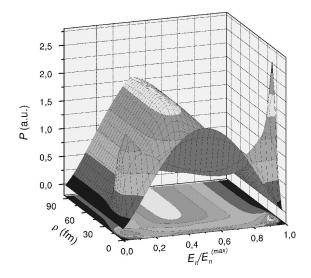


Figure 4: The energy distributions of neutrons and α -particles after decay of ${}^6{\rm He}(2^+)$ for $\theta=0.10$. The three-dimensional plot show the dependence on ρ with inclusion of 8 adiabatic wavefunctions. The maximum energies are $(m_{\alpha}+m_n)/(m_{\alpha}+2m_n)E_0$ and $2m_n/(m_{\alpha}+2m_n)E_0$ for the neutron and the α -particle, respectively. Here E_0 is the energy of the decaying resonance.

Kinetic energy distribution of third particle:

$$P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2$$

Neutrons peak at intermediate energy α -particles peak at large energy Two neutron go together, not α -neutron against neutron Virtual neutron-neutron state is essential

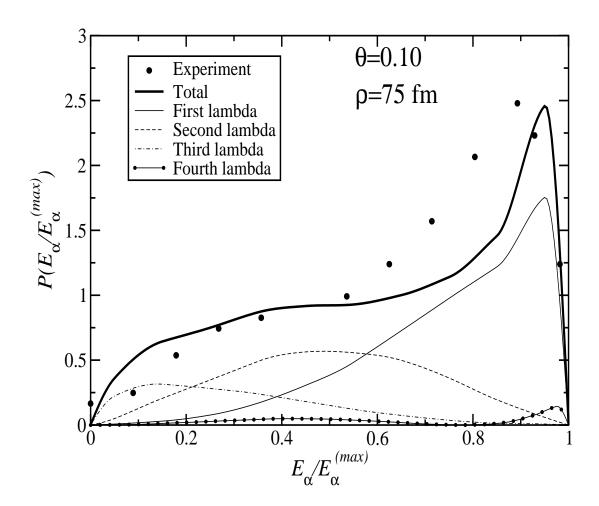


Figure 5: The energy distribution of the α -particle after decay of the 2⁺-resonance in ⁶He. The scaling angle is $\theta=0.10$ and $\rho=75$ fm where convergence is reached. The points are extracted from the measurements in [10]. Contributions from the lowest 4 adiabatic potentials are shown individually.

Old data Contributions from several adiabatic potentials Interference is important

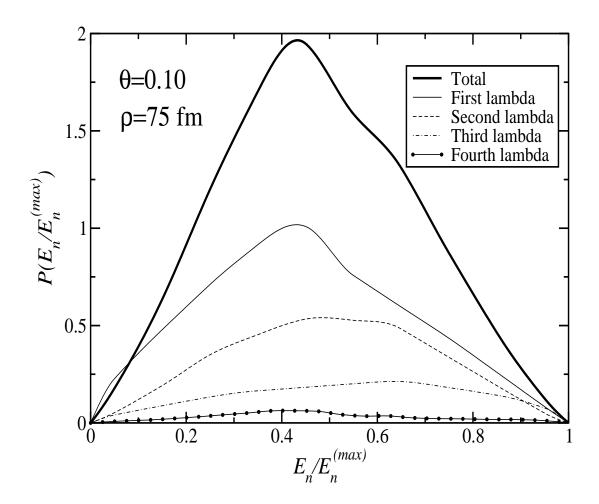


Figure 6: The energy distribution of the neutrons after decay of the 2^+ -resonance in 6 He. The scaling angle is $\theta=0.10$ and $\rho=75$ fm where convergence is reached. The points are extracted from the measurements in [10]. Contributions from the lowest 4 adiabatic potentials are shown individually.

No data Same resonance wavefunction as for α -particle

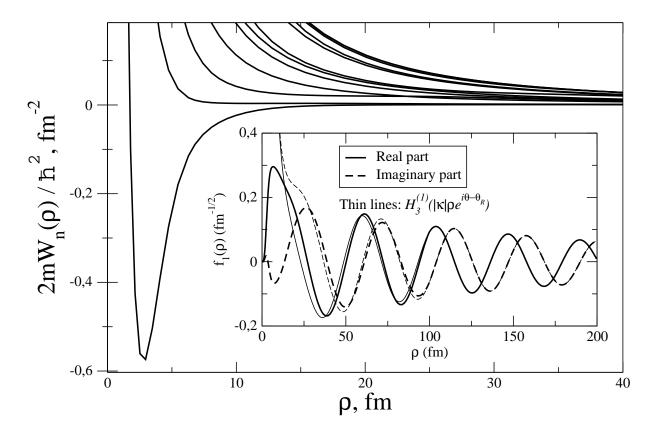


Figure 7: The lowest adiabatic potentials $W_n(\rho)$ for the $^{11}\text{Li}(1^-)$ halo nucleus within the three-body $^{9}\text{Li+n+n}$ model with interactions from [9]. The *n*-core and *n-n* scattering lengths are $a_{nc} \approx a_{nn} \approx 20$ fm. The inset shows the lowest hyper-radial resonance function with its large distance asymptotics – the Hankel function. The complex scaling angle $\theta = 0.15$, the resonance angle $\theta_R = 0.12$ corresponds to the resonance energy of about 0.4 - 0.1i MeV.

Large scattering length
Decreasing and oscillating radial function

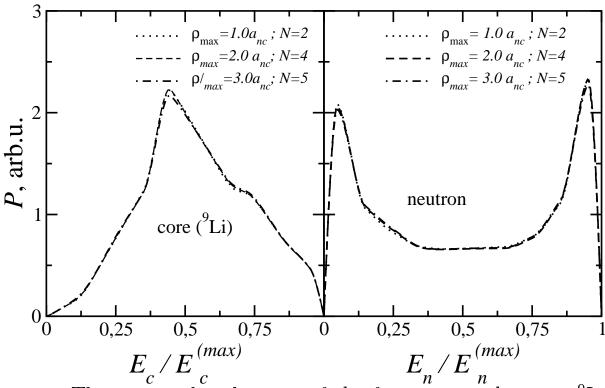


Figure 8: The energy distributions of the fragments - the core, ${}^{9}\text{Li}$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated in the three-body ${}^{9}\text{Li} + n + n$ model with only s-wave n-core interactions (scattering length $a_{nc} \approx 50 \text{ fm}$). The different curves are calculated with different ρ_{max} and different numbers of adiabatic channels N to illustrate the convergence.

Schematic model Only s-waves and large neutron-core scattering length Lowest adiabatic function is very accurate Very stable against large variation of ρ_{max} Core distribution peaks at intermediate energies Neutron distribution has a low and a high energy peak Mechanism is neutron emission, high energy Then neutron-core stick together, low energy neutron

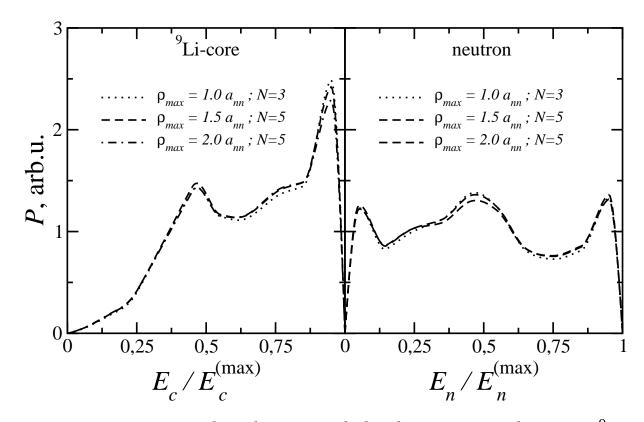


Figure 9: The energy distributions of the fragments - the core, ${}^{9}\text{Li}$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated in the three-body ${}^{9}\text{Li} + n + n$ model with s-wave n-core interactions (scattering length $a_{nc} \approx 50 \text{ fm}$), and an s-wave interaction in the n-n subsystem (scattering length $a_{nn} \approx a_{nc} \approx 50 \text{ fm}$). The different curves are calculated with different ρ_{max} and different numbers of adiabatic channels N to illustrate the convergence.

Schematic model

Only s-waves, and large neutron-neutron and neutron-core scattering length

Lowest adiabatic function is very accurate

Very stable against large variation of ρ_{max}

Core distribution gets new peak at high energy

Neutron distribution gets a new peak at intermediate energy

Two coherent decay mechanisms: Neutron emission, high and low energy neutrons, intermediate core energy

Core emission, two neutrons stick together

Core energy is maximum, neutron energy is intermediate

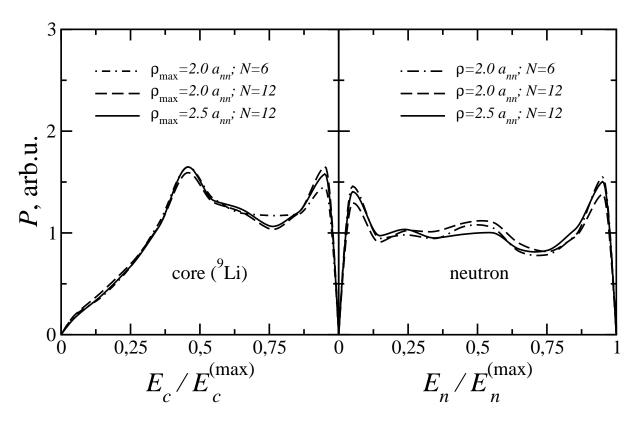


Figure 10: The energy distributions of the fragments - the core, ${}^{9}Li$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated with interactions from [9] where the *n*-core and *n*-*n* scattering lengths are about 20 fm.

Realistic interactions reproducing all other known ¹¹Li properties All three scattering lengths now about 20 fm All peaks from schematic model remains Trace of Efimov effect

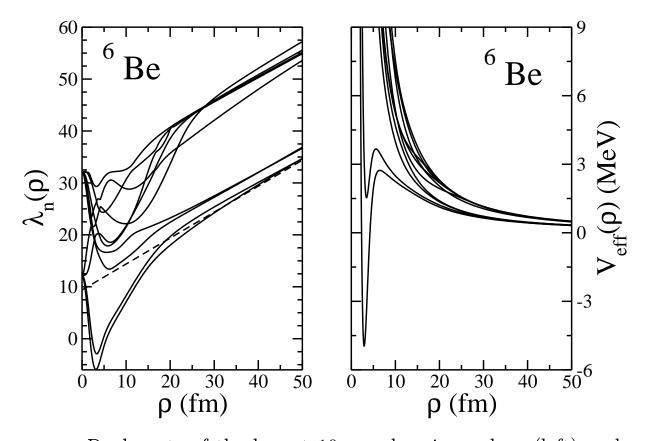


Figure 11: Real parts of the lowest 10 angular eigenvalues (left) and their corresponding adiabatic potentials (right) as functions of ρ for the 2⁺-resonance in ⁶Be. The scaling angle is $\theta = 0.15$. The dashed line is the estimated behaviour at large distances for the lowest angular eigenvalue.

The ⁶He analog 2⁺-resonance in ⁶Be Only difference is Coulomb

Small ρ : Same structure as for ⁶He

For large ρ :

Angular eigenvalues: linear in ρ , Potentials: $1/\rho$

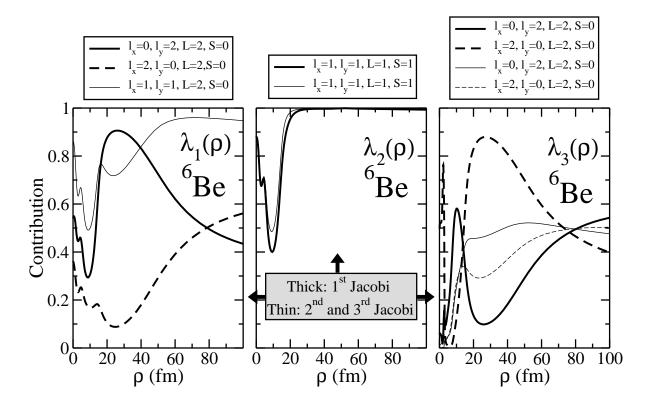


Figure 12: The fraction of the dominating components in the angular eigenfunction for the three lowest adiabatic potentials as function of ρ for $^6\text{Be}\ (2^+)$, see Fig. 1. The quantum numbers are as given in table 1. Thick lines: x refers to the two-proton system and y to its center of mass motion relative to the α -particle. Thin lines: x refers to the proton- α system and y to its center of mass motion relative to the other proton.

Partial wave decomposition of angular wavefunction Strong variation from small to large distance First eigenvalue:

Proton-proton s-wave dominates at small distance Proton-proton s and d-waves are comparable at large ρ α -proton p-waves dominate at large distance

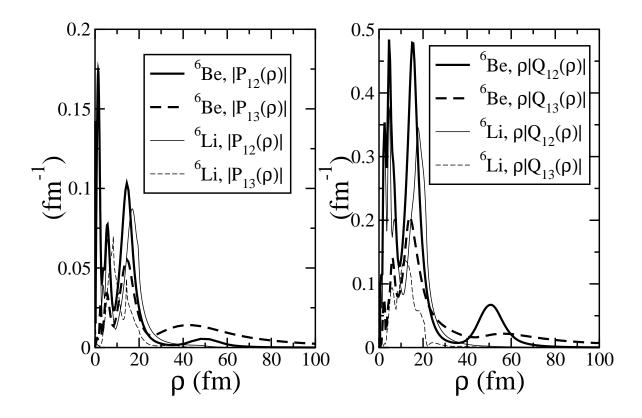


Figure 13: The absolute values of the coupling potentials between the three lowest adiabatic levels for the 2^+ -resonance in 6 Be (thick curves $\theta = 0.15$ rads) as functions of ρ , and the corresponding isobaric analog states in 6 Li (thin curves $\theta = 0.10$ rads).

Couplings determine relative size of radial wavefunctions Fall off at intermediate distance Numerical stability at large distance

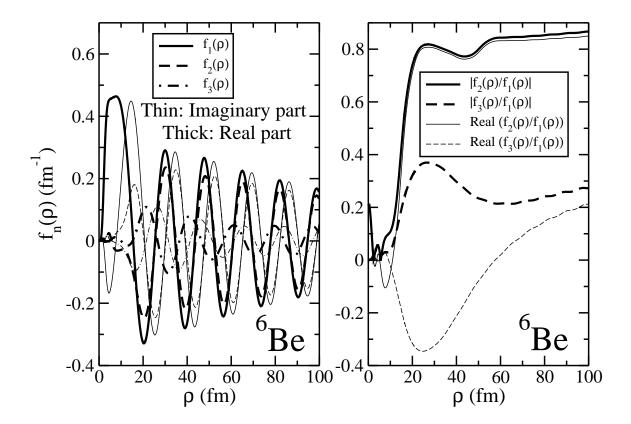


Figure 14: Left: Radial wave functions corresponding to the three first adiabatic potentials for the 2^+ -resonance in 6 Be. The real and imaginary parts are shown by the thick and thin curves, respectively. Right: Absolute values (thick curves) and the real parts (thin curves) of the ratios between the radial wave functions. The probability distribution has for each ρ been normalized to 1 as function of α .

$$\Psi(\boldsymbol{x}, \boldsymbol{y}) = \Sigma_n f_n(\rho) \Phi_n(\rho, \Omega)$$

Each fall off exponentially while oscillating around zero Relative size at large distance determine energy distribution Numerical stability is not obvious

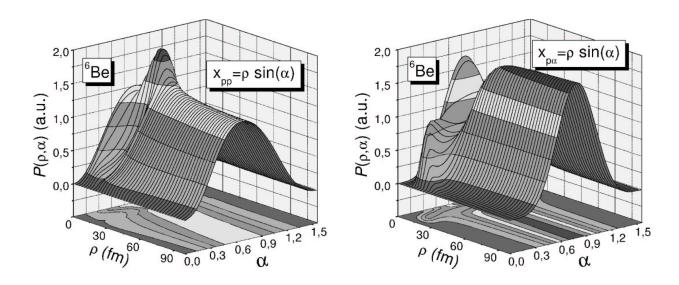


Figure 15: The probability distribution for the 2^+ -resonance in 6 Be including the lowest 10 adiabatic potentials as function of the hyperradius ρ and hyperangle α related to the distance by $r_{ik} \propto \rho \sin \alpha$, i.e. the distance between either the one proton and core r_{pc} (right) or the two protons r_{pp} (left).

Structure of total resonance wavefunction Vary strongly from small to large distance Stable at larger distance Much better than indicated by the first radial wavefunctions

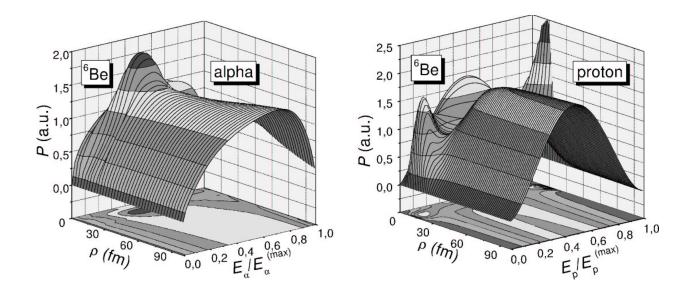


Figure 16: Kinetic energy distributions of protons (right) and α -particles (left) after decay of the 2⁺-resonance in ⁶Be. The three-dimensional plots show the dependence on ρ with inclusion of 10 adiabatic wave functions as function of $\cos^2 \alpha$, i.e. the kinetic energies $E_{\alpha,p}$ are in units of their maximum values $E_{\alpha,p}^{(max)}$ given by $(m_{\alpha} + m_p)/(m_{\alpha} + 2m_p)E_R$ and $2m_p/(m_{\alpha} + 2m_p)E_R$ for the proton and the α -particle, respectively, where E_R is the energy of the decaying resonance.

Kinetic energy distribution of third particle: $P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2$ Protons peak at intermediate energy α -particles with broad peak tilted towards large energy Coulomb is coupling a lot and broadening distributions Virtual proton-proton state is not present or very weak

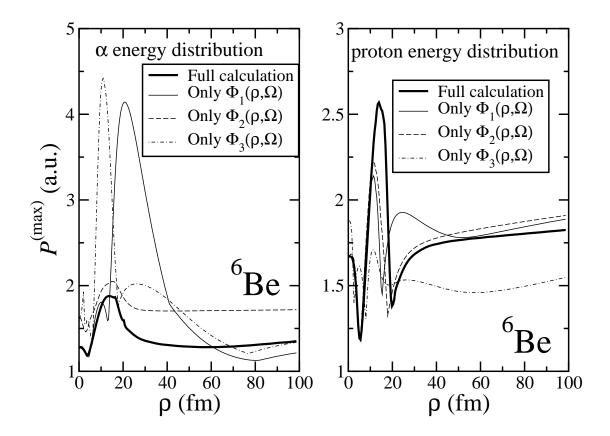


Figure 17: Projections of kinetic energy distributions for ⁶Be-decay. Thick curves: Projection of the α (left panel) and proton (right panel) kinetic energy distributions (Fig. 10) on the $E_{\alpha,p}/E_{\alpha,p}^{(max)}=1$ plane. They are then the profile originating from the maximum values of the energy distribution for each value of ρ . The thin curves are the same profile but when respectively only the first adiabatic term (solid), only the second adiabatic term (dashed), or only the third adiabatic term (dot-dashed) is included in the calculation.

The kinetic energy distribution is redistributed with ρ The total distribution is much more stable (thick curves)

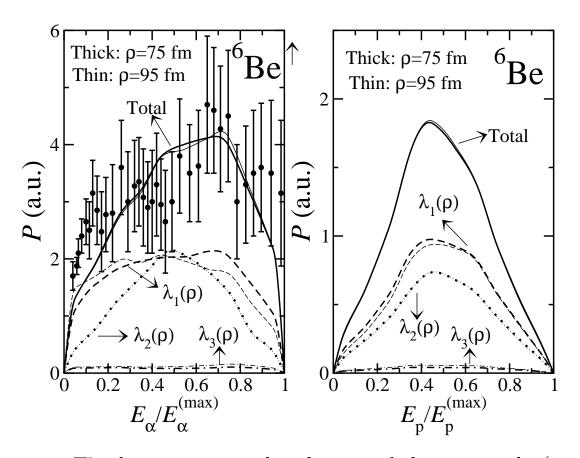


Figure 18: The kinetic energy distribution of the α -particle (upper part) and the proton (lower part) after decay of the 2^+ -resonance in 6 Be. The scaling angle is $\theta=0.15$ and the two sets of curves are for $\rho=75,95$ fm. The points are extracted from the measurements in [10]. Contributions from the lowest adiabatic potentials are shown individually.

Old data for α -particle Contributions from several adiabatic potentials

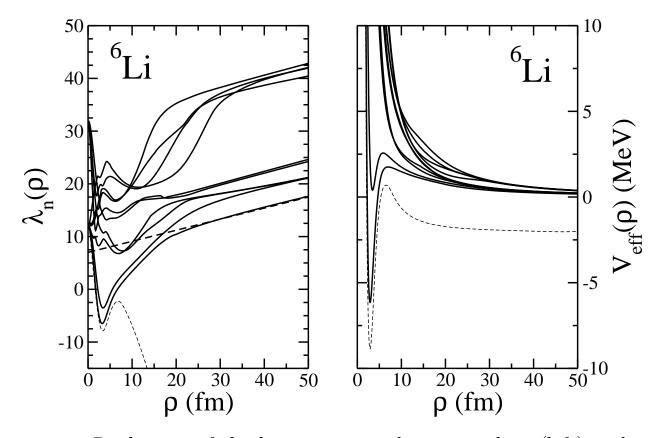


Figure 19: Real parts of the lowest 10 angular eigenvalues (left) and their corresponding adiabatic potentials (right) as functions of ρ for the 2⁺-resonance in ⁶Li. The scaling angle is $\theta = 0.10$. The dashed line is the estimated behaviour at large distances for the lowest angular eigenvalue.

The $^6\mathrm{He}$ analog 2^+ -resonance in $^6\mathrm{Li}$ Only difference is Coulomb

Small ρ : Same structure as for ⁶He

For large ρ :

Angular eigenvalues: linear in ρ , Potentials: $1/\rho$

One more state: α -deuteron structure, eigenvalue as $-\rho^2$, potential to constant

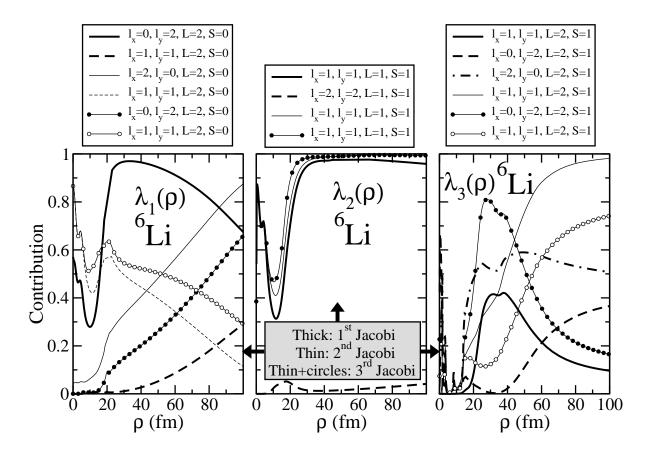


Figure 20: The fraction of the dominating components in the angular eigenfunction for the three lowest adiabatic potentials as function of ρ for $^6\text{Li}\ (2^+)$. The quantum numbers are as given in table 1. We omitted the almost decoupled lowest eigenfunction of deuteron- α character. In the second Jacobi set (thin lines) the x refers to the proton- α system and y to its center of mass motion relative to the neutron. In the third Jacobi set (thin+circle lines) the x refers to the neutron- α system and y to its center of mass motion relative to the proton.

Partial wave decomposition of angular wavefunction Strong variation from small to large distance First eigenvalue:

Proton-proton s-wave vary but dominates at all distances Proton-proton p-wave increases with ρ , isospin 0 increase with ρ α -proton d-wave dominates at large distance

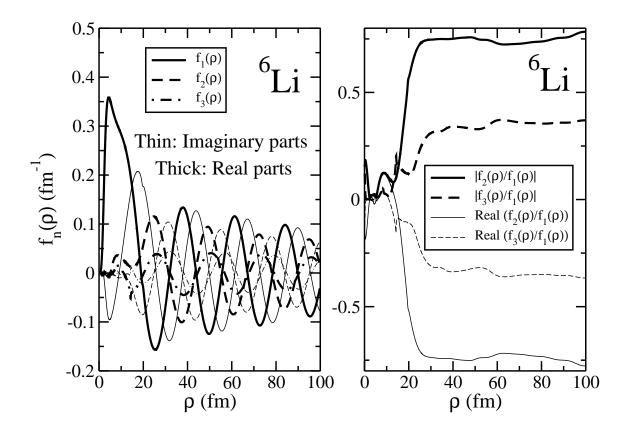


Figure 21: Left: Radial wave functions corresponding to the three first adiabatic potentials for the 2^+ -resonance in 6 Li. The real and imaginary parts are shown by the thick and thin curves, respectively. Right: Absolute values (thick curves) and the real parts (thin curves) of the ratios between the radial wave functions. The probability distribution has for each ρ been normalized to 1 as function of α .

Each fall off exponentially while oscillating around zero Relative size at large distance determine energy distribution Numerical stability is established

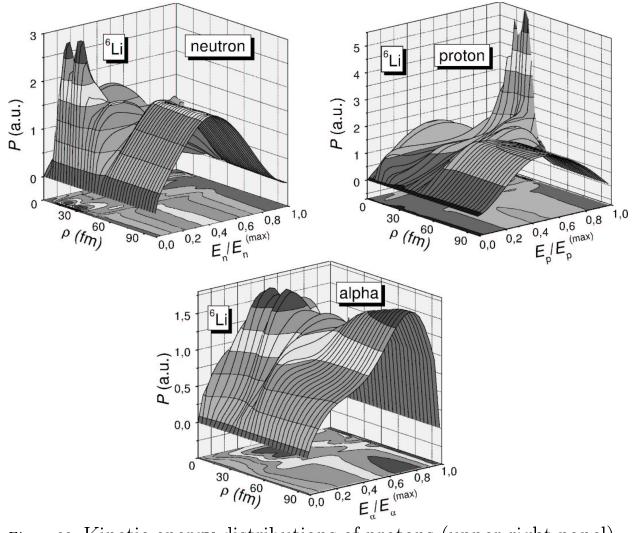


Figure 22: Kinetic energy distributions of protons (upper-right panel), neutrons (upper-left panel), and α -particles (lower panel) after decay of the 2^+ -resonance in ⁶Li corresponding to that of Fig. 10. The three-dimensional plot show the dependence on ρ with inclusion of 10 adiabatic wave functions. The maximum energies $E_{n,p,\alpha}^{(max)}$ are $(m_{\alpha} + m_p)/(m_{\alpha} + m_n + m_p)E_R$, $(m_{\alpha} + m_n)/(m_{\alpha} + m_n + m_p)E_R$ and $(m_n + m_p)/(m_{\alpha} + m_n + m_p)E_R$ for the neutron, the proton, and the α -particle, respectively, where E_R is the energy of the decaying resonance.

Kinetic energy distribution of third particle: $P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho,\alpha,\Omega_x,\Omega_y)|^2$ Protons peak at intermediate (higher) energy Neutrons peak at intermediate (lower) energy α -particles with broad peak towards large energy Neutron and proton tend to go together Virtual neutron-proton state is active Coulomb is broadening distribut? One

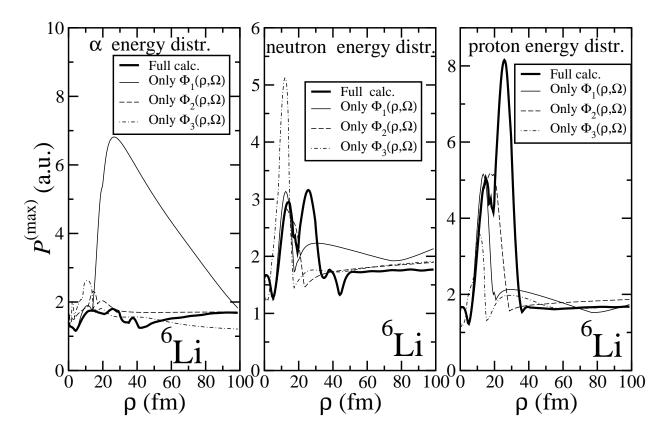


Figure 23: Projections of kinetic energy distributions as functions of ρ for ${}^6\text{Li}(2^+)$. Thick curves: Projection of the α (left panel) and proton (right panel) kinetic energy distributions (Fig. 10) on the $E_{\alpha,p}/E_{\alpha,p}^{(max)}=1$ plane. They are then the profile originated by the maximum values of the energy distribution for each value of ρ . The thin curves are the same profile but when only the first adiabatic term (solid), only the second adiabatic term (dashed), or only the third adiabatic term (dot-dashed) is included in the calculation.

The kinetic energy distribution is redistributed with ρ The total distribution is much more stable (thick curves)

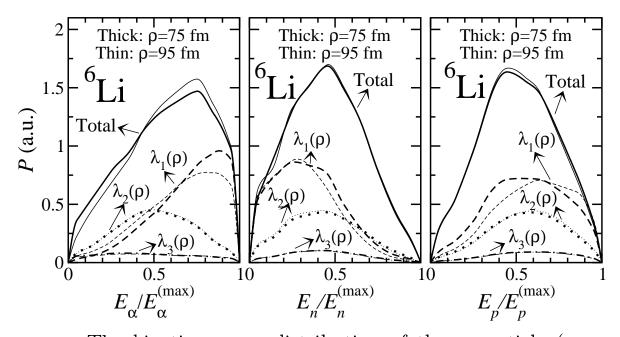


Figure 24: The kinetic energy distribution of the α -particle (upper part), the neutron (middle part) and the proton (lower part) after decay of the isobaric analog 2⁺-resonance in ⁶Li. The scaling angle is $\theta=0.10$ and the two sets of curves are for $\rho=75,95$ fm. The points are extracted from the measurements in [10]. Contributions from the lowest adiabatic potentials are shown individually.

Old data for α -particle exist in another frame Contributions from several adiabatic components Protons peak at intermediate (higher) energy Neutrons peak at intermediate (lower) energy α -particles with broad peak towards large energy Neutron and proton tend to go together Virtual neutron-proton state is active Coulomb is broadening the distributions

CONCLUSIONS

- 1. Three-body decay of many-body resonances
- 2. A model working in practice
- 3. Energies must be artificially adjusted as in α -decay
- 4. Width is estimated as hyperspherical barrier penetrability
- 5. Asymptotic wavefunction behavior determines the final state energy distributions
- 6. Asymptotics are established at intermediate distances where basis size is manageable
- 7. Large scattering lengths can be handled, Efimov effect
- 8. Coulomb can be handled in cases investigated
- 9. Isospin mixing is a dynamic effect occurring outside the range of short-range interactions
 - 10. High precision, complete kinematics, high intensity,

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DIFFERENCES TO GRIGORENKO ET AL

- 1. The spin dependence of the proton-core interaction in 17 Ne computations is in conflict with the mean-field spin-orbit interaction, i.e. the valence $d_{3/2}$ and $d_{5/2}$ states can not be independently populated, only specific combinations. Maybe corrected later on.
- 2. The hyperspherical method with only one Faddeev component is used, i.e. it is not possible to describe (i) Efimov effect, (ii) close to Efimov structure, (iii) two simultaneous 2-body substructures, (iv) one 2-body resonance unless, as he sometimes does, allow this structure as a variational degree of freedom.
- 3. The hyperspherical method has rather small $Kmax \simeq 20-25$. With three Faddeev components we need at least $Kmax \simeq 100-150$ with only the short range interaction. The Coulomb interaction and the unavoidable couplings at large distances probably increase the necessary basis size.
- 4. The three-body interaction is ρ^{-3} at large distance, i.e. excatly of the same form as the effective hyperradial potentials. This should not be necessary if the proper components are accurately included. Instead this interaction should be of much shorter range in a three-body coordinate, e.g. exponential or Gaussian.

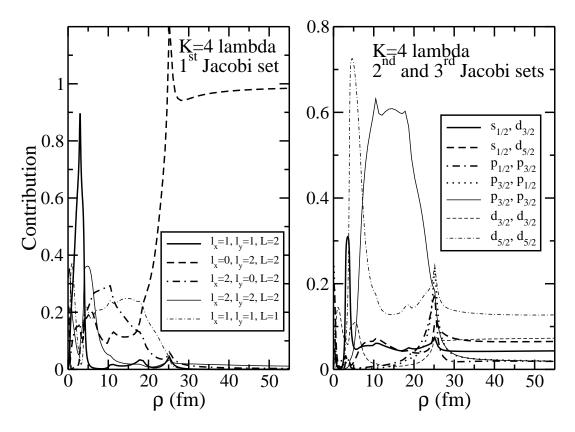


Figure 25: The fraction of different components in the fifth adiabatic potential for $\theta = 0.10$ as function of ρ for $^6\text{He}(2^+)$. The angular eigenvalue corresponds to K = 4 at large ρ , see Fig. 1. The angular momenta are specified by ℓ_x , j_x , ℓ_y , j_y , and L. Left: x refers to the two-neutron system and y to its center of mass motion relative to the α -particle. Right: x refers to the neutron- α system and y to its center of mass motion relative to the other neutron. We give the (x,y) components on the figure as ℓ_i .

Partial wave decomposition of angular wavefunction Huge variation from small to large distance Neutron-neutron s-wave dominates at large, not at small, distance α -neutron $p_{3/2}$ at small distance

All α -neutron p-waves at large distance

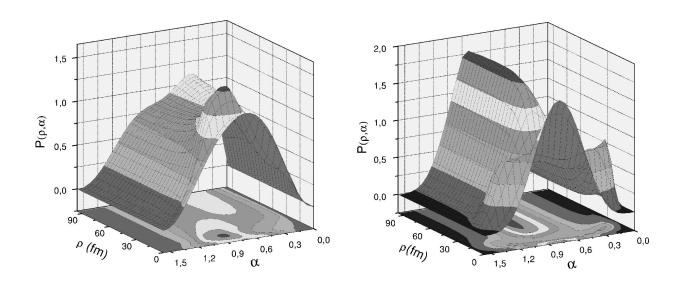


Figure 26: The probability distribution for ${}^{6}\text{He}(2^{+})$ including the lowest 8 adiabatic potentials as function of hyperradius ρ and hyperangle α related to the distance by $r_{ik} \propto \rho \sin \alpha$, i.e. the distance between either the one neutron and core r_{nc} (left) or the two neutrons r_{nn} (right).

Structure of total resonance wavefunction Vary strongly with distance Stable at larger distance

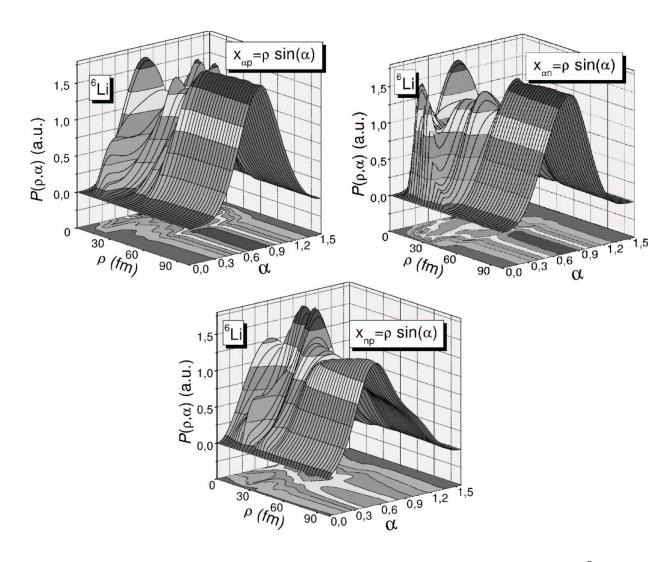


Figure 27: The probability distribution for the 2⁺-resonance in ⁶Li including the lowest 10 adiabatic potentials as function of the hyperradius ρ and hyperangle α related to the distance by $r_{ik} \propto \rho \sin \alpha$, i.e. the distance r_{cp} refers to core-proton distance in the upper-left part, r_{cn} core-neutron distance in the upper-right part, and to the r_{pn} proton-neutron distance in the lower part.

Structure of total resonance wavefunction Vary strongly from small to large distance Stable at larger distance

References

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